

Nuclear constraints on the equation of state and rotating neutron stars

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Abstract. In this contribution nuclear constraints on the equation of state for a neutron star are discussed. A combined fit to nuclear masses and charge radii leads to improved values for the symmetry energy and its derivative at nuclear saturation density, $S_v = 31$ MeV and $L = 68 \pm 8$ MeV. As an application the sensitivity of some properties of rotating supramassive neutron stars on the EoS is discussed.

1. Nuclear Constraints on the Equation of State

Despite numerous efforts to tighten the nuclear constraints on the EoS there remains considerable uncertainty. The spreading in the pressure at nuclear saturation density as summarized by Lattimer [1] a decade ago was roughly a factor six; results of present day mean field calculations vary by about a factor four [2]. Therefore it remains a challenge to try to improve the situation. The pressure as a function of density is given by $P(\rho) = \rho d\epsilon/d\rho - \epsilon(\rho)$.

In neighborhood of saturation density ρ_s , with $u = \rho/\rho_s$ and x proton fraction one has

$$\epsilon(u, x) = B + K/18(u - 1)^2 + S_A(u)(1 - 2x)^2 + \dots, \quad (1)$$

where K the compressibility and S_A the symmetry energy (SE). Hence the pressure near ρ_s is

$$P(\rho \sim \rho_s) \sim u^2 \rho_s \left[\frac{K}{9}(u - 1) + \frac{dS_A}{du}(1 - 2x)^2 + \dots \right]. \quad (2)$$

In practice the leading contribution comes from the last term, the derivative of the SE; the latter is usually parameterized in the liquid drop model (LDM) as

$$S_A = \frac{(N - Z)^2}{A} \frac{S_v}{1 + yA^{-1/3}}, \quad (3)$$

where S_v , S_s denote the volume and surface SE, and $y = S_v/S_s$.

The quantity of interest for the EoS, the derivative $L = 3\rho_s dS/d\rho|_s$, can in good approximation [1] be related to S_v and y : $y \sim 0.646 + S_v/98\text{MeV} + 0.436L/S_v + 0.087(L/S_v)^2$.

In practice the values of S_v, y when fitted to masses using the LDM appear to be strongly correlated [3], and the same is true for L, S_v (see the $1\text{-}\sigma$ confidence ellips in fig. 1), and a similar correlation is found in microscopic (mean field) models.

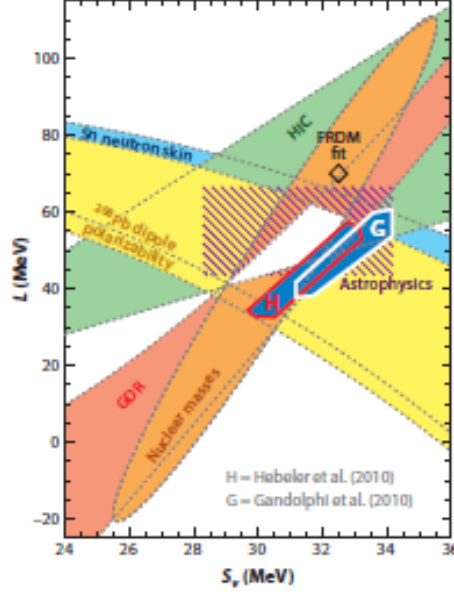


Figure 1. Constraints on L vs S_v , the filled ellipsoid corresponds to the constraint from fitting nuclear masses, from [1].

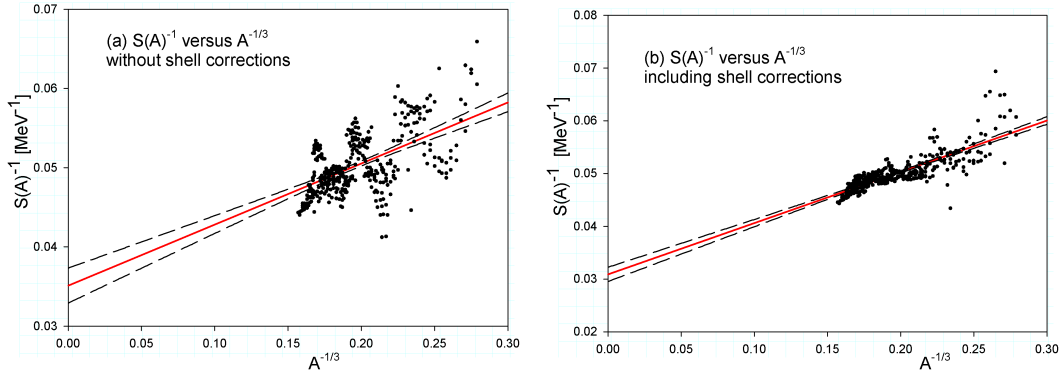


Figure 2. SE vs $A^{-1/3}$ without (left) and with (right) shell corrections, from [4].

However, one can improve the situation sketched above in several ways. As a first step one may consider differentials of masses with respect to $N - Z$ (rather than a global fit), which allows one to fit the parameters in $\frac{S_v}{1+yA^{-1/3}}$ in isolation of other terms [4, 5].

By plotting $1/S$ vs $A^{-1/3}$ (see fig. 2) one obtains the value of $1/S_v$ from the crossing of the fit line with y-axis ($A = \infty$) and the slope $y = 2.6 \pm 0.8$. From the figure the correlation between slope and S_v is evident.

As a second step an appreciable increase in the accuracy can be achieved by including shell corrections [4] shown in the right part of the figure. In passing we note that in ref. [6] a quite accurate result for S_v , S_s is reported by using double differences of masses. However, their results are obtained for a parametrization of the SE different from eq. (3), and moreover depend on the choice of the Wigner energy.

As a final step one can improve the situation further by using information from charge radii, which mainly depends only on the ratio y . (In fig. 1 this is indicated by the band labeled “skins

of Sn”, but we consider the result of this particular analysis rather model dependent). In the spirit of the LDM and distinguishing proton and neutron radii we decompose [4]

$$R_p(N, Z) = R_0(N, Z) + \frac{N}{A} R_{np}(N, Z) + \delta R_C(N, Z), \quad (4)$$

where the isoscalar term $R_0 = (NR_n + ZR_p)/A \sim r_0 A^{1/3}$, and the isovector term (essentially the neutron skin)

$$R_{np} = R_n - R_p = \frac{2r_0}{3} \frac{N - Z}{A} \frac{1}{1 + A^{1/3}/y}, \quad (5)$$

and R_C the Coulomb contribution [3]. The point is that R_{np} depends only on $y = S_v/S_s$ (apart from the Coulomb contribution).

To determine y from data one can envision the following options

(i) measure the neutron skin using parity violating electron scattering (PREX). However, the first experiment [7] on ^{208}Pb yielded a rather large error $R_{np} \sim 0.33 \pm 0.17$ fm.

(as a side remark: atomic parity violation, in progress, appears a promising alternative tool, with a possible precision of about 1% in the skin in Ra isotopes),

(ii) fit to observed charge radii using the expression (4). (As an alternative one may consider fitting differences like isobar shifts, $R_p(N, Z) - R_p(N - i, Z + i)$; the latter are independent of R_0 , but in general have larger experimental uncertainties.)

The values for S_v and y from a combined fit of masses and radii are given in table 1, which are compared to some other results from fits and microscopic approaches. Note that phenomenology

Table 1. Results for the parameters S_v , y and L obtained from fits and microscopic approaches

$S_v(\text{MeV})$	y	$L(\text{MeV})$	$R_{np}(^{208}\text{Pb})$ (fm)	ref	model
fit to masses					
32.5 ± 0.5	1.98	70 ± 15		[8]	FRLDM
32.1 ± 0.3	1.9			[6]	double diff
31.1	2.31 ± 0.38	66 ± 13		[10]	LDM
32	3.0	94		[5]	analysis IAS
31	2.5 ± 0.4	80 ± 15		[4]	LDM+shell corr
30.5	2.35 ± 0.20	68 ± 8	0.185 ± 0.015	present	masses+ charge radii
Microscopic approaches					
30		58 ± 15		[9]	Skyrme+skin Sn isotopes
32 ± 1.5		45 ± 15		[11]	QMC
31 ± 1	1.85 ± 0.25	46 ± 10		[13]	EFT
31 ± 1		64 ± 8	0.195 ± 0.02	[12]	EDF

seems to favor larger L values than most microscopic models; this trend is not understood yet.

2. Rotating supramassive neutron stars and the EoS

Several energetic observations can be associated with formation of neutron stars (NS) or black holes (BH), supernovae, gamma ray bursts (GRB). Some short GRB’s ($\sim 1\text{s}$) have been attributed to NS mergers.

Recently the observation [14] of bright radio pulses was reported, with radio flux $\sim \text{Jy}$ at GHz frequencies and $\Delta T \leq 1\text{ms}$, which do not repeat, while no γ - or x-rays were observed.

Falcke and Rezzolla [15] proposed the following interpretation: a supramassive rotating NS (i.e., a NS with a mass larger than the maximum mass of a static NS, *e.g.* created by accretion in binary system) slows down due to magnetic braking, and at critical point collapses into Kerr BH (see fig. 2). The created event horizon will hide star's surface, hence only emission from the detached magnetosphere can be observed; the estimated timescale (freefall) $\tau \sim 0.04 R_{10}^{3/2} M_2^{-1/2}$ ms appears to be consistent with the observation.

For a non-rotating star the mass vs radius relation (given the EoS) is obtained by solving the TOV equation; a rotating star requires a more general approach to general relativity. The scenario proposed in ref. [15] seems not unrealistic but some questions remain. For example the authors took a very simple representation for the EoS: a single polytrope $P = K\rho^\gamma$ with $\gamma = 2$ and K adjusted such that $M(0)_{\max} = 2.1M_\odot$.

Naturally one may ask how large is the sensitivity to EoS? To investigate this we took a 3-polytrope EoS, $P(\epsilon) = K_i \epsilon^{\Gamma_i}$. It fits $P(\rho_s)$ and has the proper low-density behavior; specifically, $\Gamma_1 = 1.5$ for $\epsilon < 67$ MeV/fm³, $\Gamma_2 = 2.68$, for $67 < \epsilon < 650$, and $\Gamma_3 = 1.41$ for $\epsilon > 650$. The K_i values are fixed by continuity of the pressure P and the normalization $P(\epsilon = 650) = 180$ MeV/fm³. It yields $M_{\max} = 2.2M_\odot$.

Using the rns code [17] the mass as a function of the equatorial radius or the central density, and the critical frequency (the Kepler or mass shedding limit) have been computed, see fig. 4. Qualitatively the main features of fig. 2 are confirmed, i.e., $M(\Omega_K)$ increases by 20%, $R(\Omega_K)$ increases by 50%. However, Ω_K itself turns out to be more sensitive to the EoS [18, 16].

Finally we point out that an observation of a high rotation frequency of a pulsar can lead to a constraint on the mass-radius diagram. Namely for a Newtonian uniformly rotating rigid star with mass M and radius R one has

$$\tau_{\min} = 2\pi \sqrt{\frac{R^3}{GM}} = 0.545 \left(\frac{M_\odot}{M}\right)^{1/2} \left(\frac{R}{10\text{km}}\right)^{3/2} \text{ms}. \quad (6)$$

Using general relativity a similar empirical relation (valid for $M < 0.9M_{\max}(0)$ with a weak dependence on the EoS) has been derived [18] (M, R refer to the static star)

$$\tau_{\min} = C (M_\odot/M)^{1/2} (R/10\text{km})^{3/2} \text{ms}, \quad (7)$$

where $C \sim 0.92 \pm 0.04$; with our EoS we find $C = 0.90$. At present, the fastest rotating pulsar has $\nu_{\text{obs}} = 716$ Hz. Obviously, the Keplerian frequency of any neutron star must satisfy $\nu_K \geq \nu_{\text{obs}}$. This inequality constraints a region on the M vs R diagram as shown in the left panel of fig. 4.

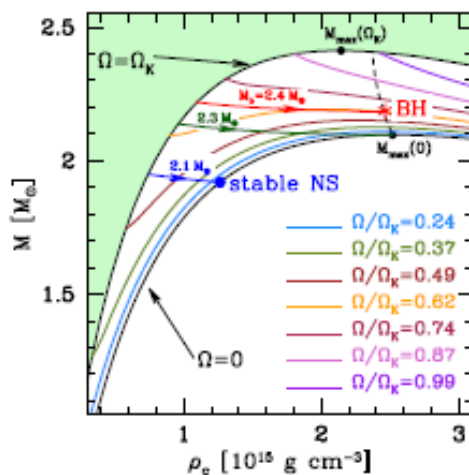


Figure 3. Gravitational mass *vs* central density for various ratios $f = \Omega/\Omega_K$, where Ω_K is taken as the critical Ω belonging to star with maximum mass; arrows represent tracks of NS's slowing down due to magnetic braking, from [15].

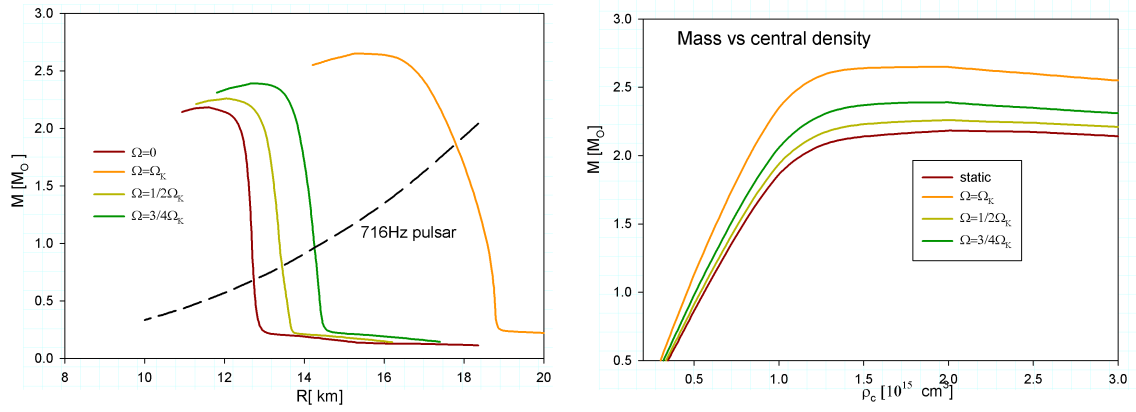


Figure 4. Gravitational mass *vs* radius (left) and *vs* central density (right) for various Ω . In the left panel the broken line denotes the lower limit on the mass of a static star from eq. (7).

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